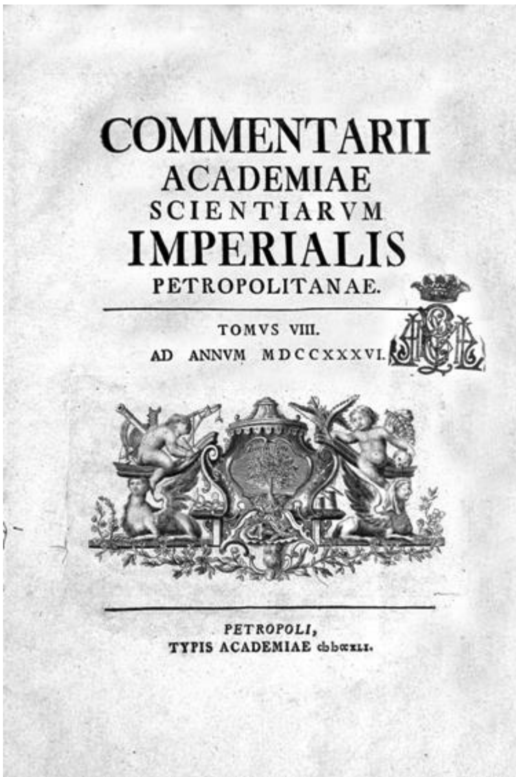
Complex Network Theory and Applications to Water Systems



Complex Network Theory (CNT) was born with Euler (1741) more than two century ago and, in the last two decades, it has become one of the most powerful and useful tool to study, describe, and understand the world (Barabási, 2012). CNT allows the study and interpretation of a huge number of systems, e.g., physical, social, biological, infrastructural, etc., since the most part of them works as network. Although each network exhibits its own topological and structural peculiarities, apparently very different networks share amazing similar features (Albert and Barabasi, 2002; Buchanan, 2003).

Research in the field of complex networks and their structural properties has grown rapidly and novel approaches, metrics and theories to explore and understand network features have been proposed (e.g., Boccaletti et al., 2006; Newman, 2010). Today this is possible thanks to the increasing computational power and availability of information for such network systems, which is the basis of the studies of complex network theory. This way, robustness, vulnerability, resilience and efficiency of real systems are increasingly investigated topics, especially with respect to the CNT and its metrics (Giustolisi et al., 2017).

Complex networks are represented as set of vertices V (nodes, crossroads, actors, sites, etc.) and connection between them as set of Edges E (links, pipes, roads, ties, bonds, etc.). For each network:

- It is possible to evaluate basic characteristics (Boccaletti et al., 2006; Newman, 2010) such as: adjacent matrix, degree, diameter, path and shortest path, walk and trial, clustering and assortativity coefficients, etc., in order of classifying their network structure and connectivity. The degree distribution, thereafter, describes the probability distribution of the number of edges connected with each vertex of the network. Several network features are associated with the shape of such distributions (Giustolisi et al., 2017). Various studies related to the classification and generation of networks defined random (Erdos and Rény, 1959; 1960), small world (Watts and Strogatz, 1998) and scale-free networks (Barabasi and Albert, 1999) with respect to their degree distribution. For random networks, the degree distribution is random and similar around an average value, following a Poisson model. Small world networks are based on the concept of six degrees of separation (Milgram, 1967; Travers and Milgram, 1969) and show a behavior between the regular and random networks (Poisson model). For scale-free networks, the degree distribution follows a Pareto model with many nodes with a low number of connection and few nodes with many connections, named hubs. The above classification is useful for analyzing the networks with respect to their emerging behavior as consequence of random failures or intentional threats.

- Some components (edges or vertices) are more central than others (Freeman, 1977; Borgatti, 2005; Bonacich, 1987, Cadini et al., 2009). In order to evaluate the most important (central) component with respect to different physical phenomena (vulnerability, robustness, etc.) some centrality metrics, based on the concept of degree, have been proposed: (1) degree (Nieminen, 1974; Freeman, 1979), (2) Closeness (Freeman, 1979 (3) Betweenness (Freeman, 1977), Eigenvector (Bonacich, 1972), Katz centrality (Katz, 1953), PageRank (Page et al., 1998), Hub and Authorities (Kleinberg, 1999), Percolation (Piraveenan et al., 2013), Cross-clique (Everett and Borgatti, 1998), Eccentricity (Hage and Harary, 1995).
- An actual issue is its division in modules, i.e. to define simpler criteria for the community detection, in order to better understanding and analysis the various portions of the network. Newman (2006) proposed an effective approach, optimizing the quality function from the CNT known as "modularity" over the possible divisions of a network. Girvan and Newman (2002) also proposed an algorithm to detect communities using a centrality metric (edge betweenness) by progressively removing edges with the highest betweenness from the original network.

New CNT-based methods are always proposed, based on connectivity features, centrality metrics, and community detection.

Several studies of CNT have focused on understanding the behavior of infrastructure networks, such as road, energy, water distribution networks (WDNs), etc. (Giustolisi et al, 2017; ; Yazdani and Jeffrey, 2011; Barthélemy, 2011; Masucci et al., 2009; Carvalho et al, 2009; Barthélemy and Flammini, 2008; Deuerlein, 2008; Rosas-Casals et al., 2007; Cardillo et al., 2006; Buhl et al., 2006; Crucitti et al., 2005; Latora and Marchiori, 2005; Buhl et al., 2004; Barthélemy, 2003) and their susceptibility to damage (Solé et al., 2017; Soldi et al., 2015; Berardi et al., 2014; Yazdani and Jeffrey; 2012a; Yazdani and Jeffrey, 2010; Berche et al., 2009; Latora et al., 2005). CNT, in fact, studies complex systems as composed of multiple interconnected components (nodes and links) structured in non-trivial configurations in which the network behavior is largely affected by the structure, depending on the organizational complexity and the level of interaction among the components (Yazdani and Jeffrey, 2011). Few researchers using the complex network approach have recently explored design, operation and maintenance of WDNs.

WDNs are infrastructure networks with special characteristics because they are city spatial networks (Barthélemy, 2011) whose topology is constrained by external environmental factors, e.g. streets and buildings, which drive their planning and construction (Giustolisi et al., 2017). This fact imposes severe limitations on network connectivity and layout, and hence WDNs are studied differently from other complex networks. The main function of such networks is to allow the efficient circulation along edges and communication among nodes within the network. Therefore, those networks need to be reliable with respect to random failures and intentional threats. For this reason, spatial networks do not generally present characteristics of scale free networks (Barthélemy, 2003).

In the last decade, the study of WDNs using the CNT has grown rapidly and has attracted many researchers, even if the first studies on the subject are previous.

Kesavan and Chandrashekar (1972) used graph theory to obtain models for the analysis of nonlinear pipe networks. Jacobs and Goulter (1988, 1989) showed that networks that are less vulnerable to failures are regular networks with equal number of links incident to each node. Ostfeld and Shamir (1996) and Ostfeld (2005) utilized network theory to study the selection of one-level system redundancy "backups" in a WDN submit to failure. Kessler et al. (1998) proposed a methodology for finding the optimal layout of a detection system in a municipal WDN based on the concept of shortest path. Gupta and Prasad (2000) proposed a numerical method based on linear graph theory for the steady-state analysis of flow and pressure in a pipe network including its hydraulic components (pumps, valves, junctions, etc.). Deuerlein (2008) proposed a new decomposition concept of the network graph according to its connectivity properties also including measures of network vulnerability and several stages of network simplification that are able to improve the understanding of the single network components and their interaction. In 2010 and 2012 Yazdani and Jeffrey represented WDNs as large sparse planar graphs with complex network characteristics and the relationship between important topological features of the network and system resilience, introducing two metrics, meshed-ness and algebraic connectivity, for quantifying redundancy and robustness, respectively, in optimization design models. In 2011 (Yazdani and Jeffrey, 2012b), they explored a variety of strategies for understanding the formation, structure, efficiency, and vulnerability of WDNs. Structural measurements are undertaken to quantify properties such as redundancy and optimal-connectivity, herein proposed as constraints in network design optimization problems. In 2012, they studied the connectivity of water distribution systems, its relationship with system robustness and susceptibility to damage, modeling weighted and directed networks. In 2011, they proposed the link-node representation of WDNs and a wide range of metrics to study the building blocks of the systems. They quantified properties such as redundancy and fault tolerance, in order to establish relationships between structural features and performance of WDNs.

Hawick (2012) applied graph theoretical analyses to some real WDNs in order to study their robustness and fragmentation properties through simulated component failure; he used betweenness centrality metric and rank network components, then removing the most important component. Gutiérrez-Pérez et al. (2013) introduced a methodology based on spectral measurements of graph theory to establish the relative importance of areas in WDNs using two popular ranking algorithms (PageRank and HITS) in order to achieve an efficient vulnerability analysis. The method is based on graph measurements such as the relative importance (ranking) and the degree of the vertices of a graph. Sheng et al. (2013) explored the formation of isolated communities in WDNs based on complex network theory using a graph-algebraic model for detecting the potential communities due to pipeline failures. Shuang et al. (2014) proposed to evaluate the nodal vulnerability of water distribution networks under cascading failures. Nazempour et al., (2016) developed a new modeling approach for the optimal placement of sensors for contamination detection in a water distribution network combining classical optimization and complex network theory. Giustolisi et al. (2017) proposed a novel metric named *neighborhood nodal degree* showing that neighborhood nodal degree distribution was suitable to classify infrastructure networks. They modeled several WDNs using the neighborhood nodal degree obtaining Poisson distributions, to means that they present a significant structural resistance to random failures and intentional threats as connectivity structure.

The graph theory concepts have been used also for the identification of the network structure of the hydraulic systems for monitoring and control purposes, as for example model calibration, metering water consumption, early contaminant detection, control of pressure and leakages (Laucelli et al, 2017). Many algorithms and metrics have been used to define the optimal WDNs division with respect to topology and asset characteristics such as pipes length and diameter, nodal elevations, leakages, etc. Torres al. (2016), for example, investigated the graph-based structural patterns and connections with engineered performance using a library of lattice-like pipe networks. Perelman and Ostfeld (2011) proposed a segmentation framework for topological/connectivity analysis with the objective of developing and demonstrating a connectivity based algorithm for WDNs analysis. The optimal segmentation and actual division of WDNs into districts is a challenging issue for analysis, planning and management purposes. A paradigm from CNT that is today largely used for WDNs segmentation is based on the modularity index (Newman, 2004). The modularity index is the most widely accepted and used metric to measure the propensity of the network division into modules (Newman and Girvan, 2004). Scibetta et al. (2013) and Diao et al. (2013) applied the modularity index to the segmentation of a WDN using the original Newman's formulation. Barthélemy, (2011) explained that the original formulation was proposed for immaterial networks. Giustolisi and Ridolfi (2014a) tailored the original modularity index in order to obtain a WDN-oriented modularity index accounting for the features of

WDNs, which are infrastructure systems. They also proposed an infrastructure modularity index (2014b), modifying the WDN-oriented modularity index, in order to overcome the resolution limit of the original modularity index (Fortunato and Barthélemy, 2007), which causes the non-identifiability of smaller modules depending on the size of the network. Finally, Giustolisi et al. (2015) reported a comprehensive framework of the WDN-oriented modularity indexes. Simone et al. (2016) extended the concepts of network segmentation to pressure sampling design, introducing the sampling-oriented modularity index and the concept of "pressure" measurement districts extending the concept of "flow" measurement districts related to the "classic" district metering areas.

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